1. Name the three major types of chi square applications. Briefly describe each.

a. Test of goodness of fit. The expected counts are completely Specified by the null hypothesis H0. Degrees of freedom equals (C - 1) Where C is the number of (columns) = number of cells altogether.

b. Test of Homogeneity. Cells arranged in rectangular R by C table. Observed counts are obtained by independently sampling pre-chosen numbers of subjects from each row which are then classified into the columns. Df = (R-1)(C-1).

c. Test of independence. Cells arranged in rectangular R by C table. Observed counts are obtained by independently sampling a fixed number Of subjects which are then classified into the table. Df = (R-1)(C-1).

1. Continued. Give examples, showing calculation of exp counts and df.

a. Test of goodness of fit. Classify 60 callers by type of service.

H_0	.2	.3	.5		
exp	12	18	30	total 60	(e.g. exp = 0.2 60 = 12)
obs	9	26	25	total 60	(found by classifying the 60 calls)

DF = (3-1) = 2.

Chi-square statistic = $(9-12)^2/12+(26-18)^2/18+(25-30)^2/30 = 5.14$.

If H_0 is correct we expect chi square ~ df = 2 (on average).

b. Test of Homogeneity. Classify 20 “day callers” by type of service. Classify 40 “night callers” by type of service. Null hypothesis is that the distribution of service request is the SAME whether day or night caller.

obs day	5	11	4	total 20 specified in advance
obs night	19	22	19	total 40 specified in advance
	24	33	23	totals from observed counts
exp day	8	11	7.67	total 20 (e.g. 24 20 / 60 = 8)
exp night	16	22	15.33	total 40 (e.g. 33 40 / 60 = 22)

Chi square statistic = $(5-8)^2/8+ \ldots + (19-15.33)^2/15.33 = 4.32$.

Df = (2-1)(3-1) = 2. If H_0 is correct we'd expect chi square statistic ~ 2.

1. Continued. Give examples, showing calculation of exp counts and df.

c. Test of Homogeneity. Sample 60 callers and cross-classify them as "day callers vs night callers" and "type of service." Null hypothesis is that day/night is INDEPENDENT of type of service.

obs day	5	11	4	total 20 "luck of draw" from obs counts
obs night	19	22	19	total 40 "luck of draw" from obs counts
	24	33	23	totals from observed counts

The analysis is just like (b).

exp day	8	11	7.67	total 20 (e.g. 24 20 / 60 = 8)
exp night	16	22	15.33	total 40 (e.g. 33 40 / 60 = 22)

Chi square statistic = $(5-8)^2/8 + \ldots + (19-15.33)^2/15.33 = 4.32$.

Df = (2-1)(3-1) = 2. If H_0 is correct we'd expect chi square statistic ~ 2.

INDEPENDENCE APPEARS AS *PROPORTIONALITY*. Obs table:

5	10	40	
15	30	120	**is a perfect example of independence.**

Exp counts = obs counts! For example, obs 5 has column total 20 and row total 55. The grand total is 220. **Exp count is 20 55 / 220 = 5**.

2. Determine P-value for chi square statistic = 15.81 with df = 6.

df	P-value		
	0.015	0.0149	**0.0148**
6	15.78	15.79	15.81

3. Interpret (2). If H_0 is correct there is around 0.0148 probability that we'd see a chi square statistic at least as large as our 15.81.

Either H_0 is wrong or we've seen an event of rarity 0.0148.

We may never know which is the case.

Statistics evaluates the rarity of getting at least as much evidence against H_0 as we have seen from our data if indeed H_0 is correct.

4. Automated decisions based on P-value. We can choose some Threshold and use it as a criterion or rejecting H_0. For example, if we decide to reject H_0 when P-value < 0.001 then we will (by this policy) reject H_0 with probability 0.001 in a case in which H_0 is true.

5. Sampling to a foregone conclusion. If H_0 is NOT CORRECT then a chi square test is nearly certain to find that out with enough data.

For example, consider a test of

H_0: 10% defective product

H_1: not 10% defective product

Based on a sample of n items from production.

If 10.0000001 percent of product is defective we will be nearly certain to reject H_0 if n is very large.

A chi square test with lots of data is like an overly sensitive smoke detector. It will scream "fire" if even a light departure from the type of data associated with H_0: "no fire" is detected.

6. Meta Analysis (merging chi square from INDEPENDENT experiments).

experiment A	chi square statistic = 15.67	df = 7
experiment B	chi square statistic = 6.94	df = 2

Define H_0: H_{0A} AND H_{0B} are BOTH correct.

If H_0 is correct then the sum of the chi square statistics is chi square Distributed ith df equal to the sum of dfA and dfB.

For this combining of the two experiments we have observed a chi square statistic of 15.67 + 6.94 = 22.61 with df = 7 + 2 = 9.

Consulting the chi square table find P-value ~ 0.0071:

	0.0071
9	22.62

For experiment A alone P-value is off table (> 0.015).
For experiment B alone P-value is off table (> 0.015).

7. Df = the number of parameters in the full model minus the number of parameters in the H_0 model.

a. Hardy-Weinberg: Under **random** mating, the probabilities of gene types AA, Aa (includes aA), aa are p^2, 2 pq, q^2 , where p and q are the fractions of letters A and a in the mating population. If the parent population has 30% A and 70% a letters to give up, the offspring population will look like 9% AA, 42% Aa and 49% aa. In the full model here are (3-1) parameters. Under the random mating hypothesis there is only one parameter p. So df = (3-1) - 1 = 1.

b. Homogeneity. Row totals are specified in advance of sampling. The full model thus has (C-1) parameters in each row, for a total of R(C-1). Under homogeneity hypothesis there are only C-1 parameters. Therefore df = R(C-1) - (C-1) = (R-1)(C-1).

c. Independence. A random sample of subjects are classified into the R by C cells of table. The number of parameters in the full model is RC-1. Under the hypothesis of independence there are only (R-1)+(C-1) parameters needed. So df = (RC-1) - (R-1) - (C-1) = (R-1)(C-1).

This results in the same df as in (b).